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REMARKS ABOUT R. P. FRASER'S REPORTS ON JET RESEARCH

by K.O. Friedrichs

A Memorandum Submitted
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to the
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INTRODUCTION

The New York University Group of the Applied Mathematics Panel is grateful to the Liaison Office of the O.S.R.D. for providing a copy of R. P. Fraser's work, completed in the years 1940-41, which was needed in connection with a request from the Bureau of Aeronautics. The following analysis of Fraser's results (by K. O. Friedrichs) was carried out, in the light of more recent shock theory, in the hope of making these results useful for application to various problems of supersonic flow.

R. Courant

REMARKS ABOUT R. P. FRASER'S REPORTS ON JET RESEARCH

The object of the present memorandum is to discuss and to interpret experimental results presented by R. P. Fraser* in a series of most remarkable shadow photographs of the jet flowing out of exhaust nozzles and orifices at high speed. These photographs, it seems, can be deciphered more completely now than had been possible in the years 1940-41 since, in the meantime, a deeper theoretical understanding of possible patterns of shock and expansion waves has been attained. On the other hand, a proper interpretation of experimental results such as those by Fraser might be important for the investigation of phenomena inside exhaust nozzles of rocket motors and other devices.

Under this point of view those results by Fraser will be analyzed here which seem connected with matters of some actuality.

The first report (July 1940) concerns flows through nozzles involving various kinds of changes such

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*Fraser, R. P.: Flow through Nozzles at Supersonic Speeds - Four interim reports on Jet Research, July 1940 to June 1941.

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as a grid inscribed in the nozzle, a relatively long parallel section at the throat, a co-axial ledge at the throat, nozzles whose divergent and convergent parts are out of line. The result is that all these changes are not seriously detrimental to the jet. Also multiple nozzles are investigated; they involve considerable losses. (Photos 78 to 110 belong to this series). Photo 78, e.g., shows very clearly the Mach lines in the jet resulting from slight disturbances at the nozzle wall.

Of more specific interest for us is the second report (Oct. 1940 and Jan. 1941). A set of nozzles with half-angles of divergence from 5° to 30° are investigated. All these nozzles have the same throat area and the same exit area (in ratio 1 to 6). These areas correspond, on the basis of the hydraulic theory, to a ratio 60 to 1 of chamber to exhaust pressure; i.e., if the chamber pressure is 60 atm then one expects the jet to have expanded to just 1 atm at the exit. The contour line of the nozzle consists of two sections. The convergent entry section is an arc of a circle which just attains axial direction at the throat; the divergent section of the contour is a straight line; (i.e., the divergent section of the nozzle is conical). This design was chosen for simplicity in manufacturing and adjustment; it implies, of course, a

considerable change in direction at the throat (from 5° to 30°). Air under pressures ranging from 1200 to 200 lb/in^2 (i.e., 80 to 13 atm) was fed to the chamber and the resulting jet emerging from the nozzle mouth was photographed. The jet was apparently steady.

For the interpretation of the photos, the following considerations are essential. Suppose a conical surface (or any surface of revolution) encloses air of greater density than that of the air outside. (See Table 1) Then light rays crossing perpendicularly to the axis will be so deflected that the image of the cone on the photograph consists of an unexposed (white) strip toward the inside of the cone contour followed by an overexposed (dark) region.* This applies to shock fronts facing <u>outward</u> (i.e., across which the air <u>enters</u> the interior of the cone) and to contact surfaces (i.e., stream surfaces separating regions of different density). The same applies to rarefaction wave regions when the air stream crosses them from interior to exterior; since rarefaction waves spread and do not represent sharp discontinuities the effect is less pronounced.

If the density inside of the cone is less than that outside, the image consists of an overexposed (dark) strip at the outside and an underexposed (light) region

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*Cf. Hilton, Proc. Roy. Soc. A 169, 174 (1938)
H. Lewy; B.R.L., Aberdeen, Rep. No. 373 (143)
Ph. C. Keenan, Expl. Res. Rep. No. 11 (Re2c)
Navy Dept. Bureau of Ord. Wash. (Rest.)

toward the inside of the cone contour. This applies to shock fronts facing inward; i.e., across which the air <u>leaves</u> the interior of the cone, and to contact surfaces.

A common feature of all the photographs of jets coming out of nozzles is a sharp white strip beginning at the nozzle rim. This strip is due to the fact that the boundary of the jet is a contact surface and it indicates that the density in the jet is greater than that of the outside air. (Greater density of the jet would imply lower temperature since the pressure is the same at both sides of the jet boundary. Probably the chamber temperature was not excessively high; in that case, the jet temperature should be very low indeed). As is expected, the discontinuity at the jet boundary develops into a boundary layer which gradually disintegrates; vortices are formed and the flow becomes agitated. This is indicated rather definitely in the photographs.

When the chamber pressure is greater than 900 lb/in^2, for which the nozzle was designed, the pressure in the jet at the mouth will be greater than one atmosphere. Hence, a rarefaction wave across which the pressure is adjusted to the atmosphere is expected to start at the nozzle rim. Such a wave seems indicated as a white strip in photo 48 (Table 2) which shows the jet coming out of the 5° nozzle (No. 24) (Table 1) working with 1170 lb/in^2 chamber pressure.

The rarefaction wave disappears when the mouth pressure is about 1 atm; and when the mouth pressure has fallen below 1 atm, a shock front is needed for adjustment. This is the situation for the flow shown in photo 49 (Table 2) corresponding to 800 lb/in^2 chamber pressure. A weak conical shock front begins at the rim which is "reflected" as a second conical shock front. It is interesting to note that the white strip which indicates the reflected shock front shows up much clearer than the light region preceding the dark line which indicates the "incident" shock front. This is in agreement with the predictions from the optical considerations made before. Photo 50 (Table 2) corresponds to 485 lb/in^2 chamber pressure. The incident shock front beginning at the rim, the "rim shock", is stronger and more inclined. The incident and reflected shock cones are truncated; the reflection does not take place at a point but through a "Mach shock front", a disk perpendicular to the axis. The cylinder of subsonic flow past the Mach disk is seen in the figure. The reflected shock is in its turn reflected at the jet boundary through a rarefaction wave, which is faintly indicated in the photo. When the chamber pressure is 214 lb/in^2, the jet has detached somewhere inside the nozzle as is clearly seen in photo 51 (Table 2).

It is interesting to observe that the jet attains its minimum cross-section where the reflected shock front is reflected. The flow through this cross-section behaves somewhat like the flow across a nozzle throat or rather out of an orifice; the flow changes from subsonic to supersonic speed.

The flow pictures are similar for the 10° nozzle. Photos 43 and 44 (Table 3) for the flow out of a 15° nozzle (No. 27) exhibit new phenomena. For 875 lb/in^2 chamber pressure, one should have about 1 atm at the mouth and expect only a very faint shock or rarefaction wave to begin at the rim. A faint wave of this type is seen to begin near the rim; it develops into a shock-wave which becomes stronger when it approaches the point of reflection, and the reflected wave has considerable strength, a most peculiar phenomenon which we shall observe more clearly later on.

Another peculiar phenomenon is seen: A diverging weak shock front emerges from the interior of the nozzle. It is not difficult to explain its origin. The sharp change in direction at the throat forces the jet to expand suddenly there. This expansion will be stopped by a shock front, the "throat shock". The throat shock will be reflected inside the nozzle and

a part of this reflected throat shock extends to the outside of the nozzle. A similar throat shock pattern should occur in the 5° and 10° nozzles; but either these shocks are too weak to be observed or they have faded out in these long nozzles.

When the nozzle is shorter (or the divergence angle larger) the throat shock pattern should extend further out of the nozzle, and both incident and reflected shock should be visible. Strangely enough, this is not, or at most only faintly, seen in the 20° nozzle (No. 33), (Photos 39, 40, 41, 42), but it will be seen in Photos 35 to 38 discussed below. (Table 4)

In Photo 35, depicting flow out of a 22.5° nozzle (No. 32) with high chamber pressure 1030 lb/in^2, one can spot a rarefaction wave beginning at the rim. This wave is evenutally stopped by a shock front of gradually increasing strength, the "stopping" shock front; the reflected "stopping" shock front is of considerable strength. In Photo 38, which corresponds to the low chamber pressure of 240 lb/in^2 one observes that the jet has already detached from the wall in the interior of the nozzle. This becomes clear when one determines the position of the rim by comparison with a flow under high chamber pressure, Photo 35 say. Also, the boundary layer has already started

disintegrating and the rim shock comes clearly from the interior of the nozzle.

The throat shock can be observed very clearly in Photos 35 to 38. The reflected throat shock is seen faintly in Photo 36. Photo 37 presents a very interesting flow pattern. The chamber pressure (425 lb/in^2) and hence also the mouth pressure are smaller than in the flow considered just before. The smaller pressure apparently forces the rim shock to curve inward more strongly and the reflection to take place nearer to the mouth. The throat shock is affected in such a way that it suffers a Mach reflection. The reflected throat shock soon intersects the Mach disk of the rim shock; for lower pressures (Photo 38) the Mach disk of the throat shock nearly merges with the Mach disk of the rim shock.

There is no doubt that the incident throat shock is weak while the reflected throat shock is strong; this follows from the fact that the angle of the incident throat shock with the axis is small while that of the reflected throat shock is large. This fact is of considerable interest. It is known that a given incident shock can be reflected, if at all, in two ways, through a weak and a strong shock. For progressing shock waves, the strong reflection has been rejected. It is inter-

esting to see that for the steady flow out of a nozzle, which is the result of adjustment to a stable position, strong reflection is possible. It is likely, too, that the reflection of the rim shock (or the stopping shock) is of the strong variety, at least as long as the reflection is of the Mach type.

In Photos 31 to 34 depicting the flow out of the 30° nozzle (No. 31) one can just see the throat shock coming out of the nozzle (Photo 31) and its Mach disk which has nearly merged with the Mach disk of the stopping shock. In Photo 34 the Mach disk of the rim shock appears in a strange manner. It would probably be possible to explain this figure by unraveling the paths of light rays passing the neighborhood of this disk. Jet detachment in the nozzle is indicated for 370 lb/in^2 chamber pressure (Photo 34).

Our actual interest in Fraser's report derives mainly from his pressure measurements. The discussion of the flow pattern is to serve mainly for a proper interpretation of these measurements.

A hole was drilled through the nozzle wall near the mouth and the pressure of the air stream at the wall was measured. (We shall refer to the pressure at the point of tapping as "test point" pressure). It was found that the pressure ratio; i.e., the ratio of chamber pressure

to the test point pressure was nearly constant for chamber pressure ranging from 1100 to 500 lb/in^2. (The average value of this nearly constant ratio will be called the average experimental pressure ratio). For smaller chamber pressures, the test point pressure increased rapidly. This was explained as indicating that jet detachment occurs before reaching the test point. The mouth pressure at which jet detachment first occurs is found to lie between .5 atm and .65 atm rather independent of the divergence. This is a remarkable result. One might have expected that jet detachment would begin at lower mouth pressures for long nozzles and at nearly 1 atm for short and strongly diverging nozzles.

The measured values of the pressure ratios are compared with those calculated on the basis of the hydraulic or one-dimensional theory for nozzle flow: (for this calculation one need only know the chamber pressure, the throat area and the cross-section area at the point of tapping). At first, one should expect that the experimental value of the test point pressure is greater than the calculated value, at least for nozzles with considerable divergence, since divergence offers to the air stream the opportunity of quick expansion. Certainly this would be so for shockless and frictionless flow through a smooth nozzle . Fraser's experimental results

however, show just the opposite, except for the 15° nozzle, as is seen from the table for the calculated pressure ratio R_{calc} and the average experimental pressure ratio R_{exp}. The table is copied below except that the values of R_{calc} and R_{exp}/R_{calc} were recalculated. (See also Table 5)

Nozzle	Half angle	R_{calc}	R_{exp}	R_{exp}/R_{calc}
31	30°	53.59	48.7	.91
43	25°	52.10	43.7	.84
32	22.5°	54.43	45.6	.84
33	20°	58.17	50.4	.87
27	15°	58.17	63.2	1.09
27a	15°	54.86	57.6	1.05
26	10°	58.87	57.9	.98
24	5°	57.53	56.05	.97

In interpreting these results one should not forget that the nozzles tested by Fraser involved a sudden change of direction at the throat. As a consequence of this disturbance, a shock front had developed, which was observed in some photos, corresponding to nozzles of 15° and wider opening. On crossing such a shock front the pressure increases suddenly and it may well be that this fact is responsible for the major portion of the 10 to 15 °/o increase in test point pressure observed in nozzles Nos. 31, 32, 33 according to the table above. In the some-

what longer 15° nozzle (No. 27), the flow past the shock has apparently had enough chance to expand and thus to decrease the pressure again; that the result amounts to 5 to 7°/o more than the calculated pressure remains strange. (A deviation of a few percent is of no significance since the measurements cannot be expected to be more accurate than 2 or 3°/o.) Whether or not the occurrence of shocks is mainly responsible for the deviation from theoretical expectation is still the question. Fraser himself gives a different explanation: "At 15° and greater divergences it may be that the frictional loss in the convergent portion is the more important part. This causes a consequent drop in the pressure at the throat to below the theoretical critical pressure and, therefore, a consequent drop in the pressures along the divergence. The rising mouth pressure with increasing divergence must then be attributed to a further departure from the adiabatic expansion in the divergence due to general turbulence rather than wall friction." Fraser's argument would also hold for nozzles with a smooth throat. It would, therefore, be of interest to have experimental evidence about the performance of smooth nozzles with wide divergence.

The preceding remarks refer to the pressure at the wall. The flow distribution over the cross-section is certainly influenced considerably by the shock fronts.

I. W. Maccoll, in a note added to Fraser's interim report for February and March 1941, observes that better agreement with the result of the hydraulic treatment could be expected if, for comparison, the average pressure over a cross-section is taken, instead of the wall pressure. He points out that the photographs for the flow through the 15° nozzle show that the pressure near the axis is greater than at the wall. This is also clear from the fact that the air passes the observed shock front from exterior to interior. The ratio R_{exp}/R_{calc} would, therefore, be less than 1.07 if R_{exp} were based on an average over a cross-section. For the 20° to 30° nozzles the direction of the throat shock is such that the pressure in the interior should be less than at the wall, and, consequently, the ratio R_{exp}/R_{calc} should increase when based on the average over a cross-section.

In any case, Fraser's pressure measurements are not sufficient to derive conclusive information on the performance of smooth nozzles with wide divergence.

A few indications may be given about the contents of the third and fourth report. The experiments were repeated with a new 15° nozzle (No. 27a) and the previous strange results were confirmed. A smooth 10° nozzle (No. 41) was tested and the results were nearly identical with those for the 10° nozzle (No. 26) with change of direction at the throat

previously tested. It would have been interesting if smooth nozzles with wider divergence had also been tested. A 10° nozzle (No. 42a) with a shorter entry section was tested (Photo 55, Table 6); it is interesting that in this case a reflected throat shock becomes visible. This shows that the entry portion has an influence on the flow pattern even in a long nozzle. A similar nozzle (No. 42) with a parallel section at the rim was tested (Photo 56, Table 6). One expects that a shock front of some strength will develop at the beginning of this parallel section and that the pressure at the wall of this section rises considerably. Both expectations are clearly confirmed.

In the fourth report* a nozzle designed for the rather low chamber pressure of 80 lb/in^2 = 5.44 atm was tested (Photos 3 to 8, Table 7). For 30 lb/in^2 the shock front has just come out of the nozzle, (Photo 3). For 80 lb/in^2 the emerging stream is rather parallel and only a weak wave develops from the rim, (Photo 6). For greater chamber pressures (100 and 160 lb/in^2) rarefaction waves develop from the rim which are particularly visible. Again these rarefaction waves are stopped by a shock front of gradually increasing strength; this stopping shock is then reflected by shocks of apparently considerable strength.

*Another problem treated in this report may be briefly mentioned: Obstacles were opposed to the jet, the resulting flow pattern was photographed, and the sometimes very strange back reactions on the nozzle were measured.

A gradually developing stopping shock with a rather strong reflected shock is particularly clearly seen in photo 1 (Table 6) depicting the flow out of an orifice; i.e., a nozzle which consists of only a diverging section, the exit being at the throat. If the pressure is increased, (Photo 24, Table 6), the stopping shock begins nearer the rim and becomes rather strong; the Mach disk nearly covers the whole jet cross-section.

The flow patterns in the jet as revealed by Fraser's Shadowgraphs differ decisively from the flow pattern described by Prandtl (1907). Prandtl's pattern for a jet which emerges from the nozzle with a pressure greater than atmosphere is easily described for two-dimensional flow emerging with constant velocity. Two rarefaction waves begin at the two edges through which the pressure adjusts to atmospheric pressure; these waves, after intersecting each other, are reflected at the opposite boundaries of the jet as contraction waves, which eventually converge to two points on the original boundaries; this flow pattern then repeats itself periodically. (See Table 8) It was apparently Prandtl's opinion that for jets emerging with non-constant velocity and for three-dimensional jets, the flow pattern would be similar. Stanton (1926) had concluded from various experiments that the actual flow pattern is quite different.

Fraser's Shadowgraphs show the deviation of the actual pattern from Prandtl's pattern with particular clarity. V. Neumann, who in this connection had called the writer's attention some time ago to Fraser's photographs, had pointed out that the decisive new (and unexpected) element in the correct flow pattern is what was here called "stopping shock" of a rarefaction wave; that considerable over-expansion occurs in the front of the stopping shock (and the Mach shock); and that the stopping shock cuts through the Mach lines composing the rarefaction wave, beginning at the last of these lines in the direction of that line with strength zero. (See Table 8).

This strange phenomenon calls for an explanation. It is rather instructive for this purpose to compare Photos 48 and 31 which show the jet produced by essentially the same chamber pressure, 1140 and 1150 lb/in^2. Although the exhaust pressure is approximately the same in both cases, approximately 1.3 atm, the flow patterns are quite different. Stopping shock and reflected shock are much stronger in the jet with greater divergence; and it becomes clear that divergence has a strong influence on this phenomenon. It seems that the phenomenon can be explained as due solely to divergence, and the fact that atmospheric pressure must prevail at the jet boundary. It is evident that in a diverging flow the pressure will continue to decrease and very rapidly at that. The reason is that the stream will continue in

the same way that it would if the nozzle continued, until it reaches the first Mach line of the rarefaction wave. Across the rarefaction wave the pressure decreases further and it is thus clear that there will be a pressure gradient from the jet boundary toward the inside of the jet. This pressure gradient then will cause the jet to turn inward. Since the pressure is constant along the jet boundary, the Mach angle there is also constant. As a consequence, the Mach lines issuing from the jet boundary under the Mach angle turn inward. Although it is not excluded by the present argument that these Mach lines eventually diverge again, it appears that they do not, but rather intersect. It could be seen that the first point of intersection; i.e., the tip of the envelope of these Mach lines, lies on the external Mach line of the rarefaction pencil. Thus it follows that a shock front must begin somewhere past the rarefaction wave and cut into it. That the shock should begin at the rim is not indicated; various of Fraser's flow pictures could be so interpreted that it begins later. The arguments advanced could be corroborated and refined by considering the hodograph of the flow.

In this connection, a paper by J. Hartmann and F. Lazarus should be mentioned (The Air-Jet with a Velocity Exceeding that of Sound, The Philosophical Magazine, XXXI, January, 1941), in which many photographs of jets from an orifice are shown and pressure measurements are reported. It was found that the pressure within the jet decreases strongly to a value far below atmospheric pressure until raised by a shock front. A feature of particular interest can be seen from the photographs: when the chamber pressure is small, a stopping shock begins near the opposite boundary of the jet. In that case the shock front consists only of a truncated cone, which corresponds to what had been called "reflected" shock front. As the chamber pressure is increased, the truncated cone becomes a full cone and, on further increase, the Mach disk and what had been called the "incident" shock front appear. It is clear that the "incident shock" is weak shortly after it has appeared, and on this basis it could be seen that the reflected shock is of the strong variety. These results seem to corroborate the arguments given above.

The latter phenomena did not occur in Fraser's shadowgraphs. There is no doubt, however, that on closer scrutiny they would reveal much additional information on possible jet patterns.

TABLE 1

NOZZLES OF VARIOUS DIVERGENCES

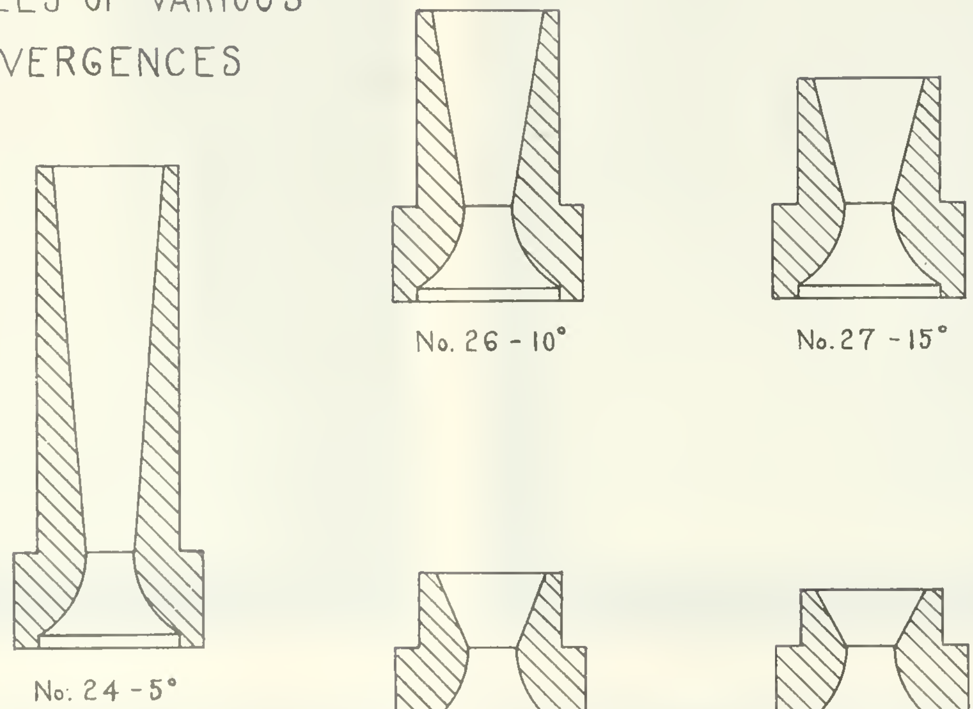


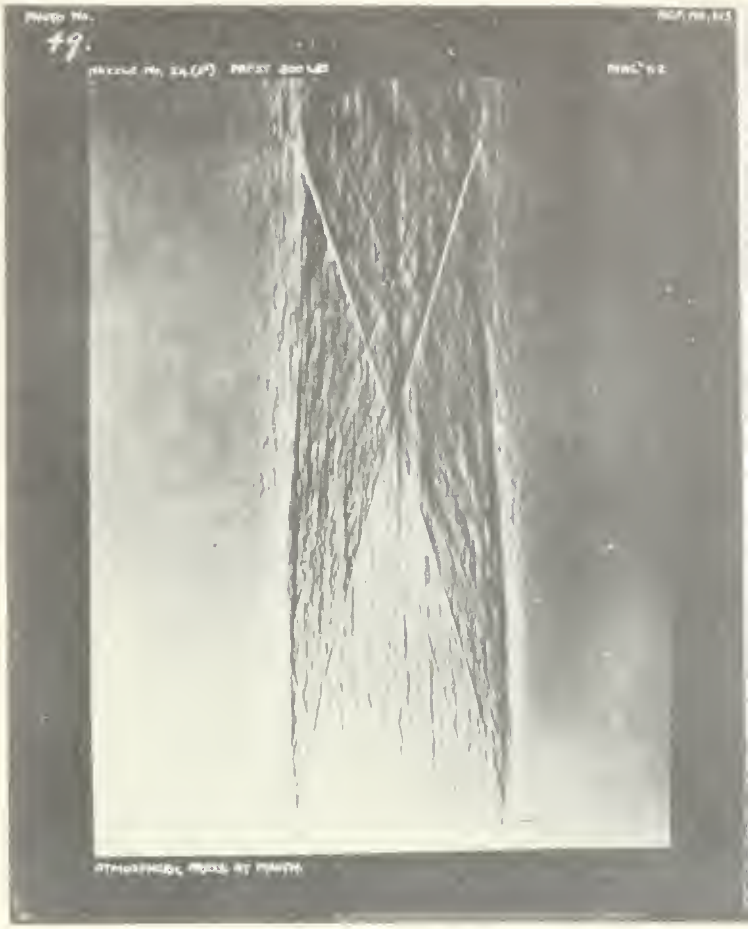
IMAGE OF CONE OF GAS

Cone of gas

Image of cone when interior is more dense than exterior

Image of cone when interior is less dense than exterior

TABLE 2



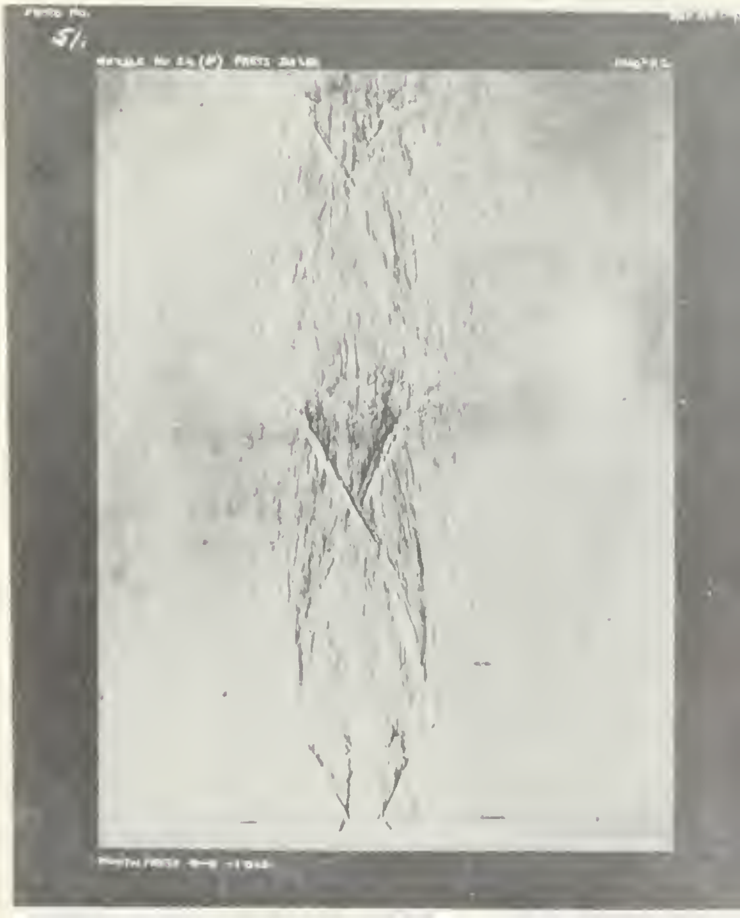
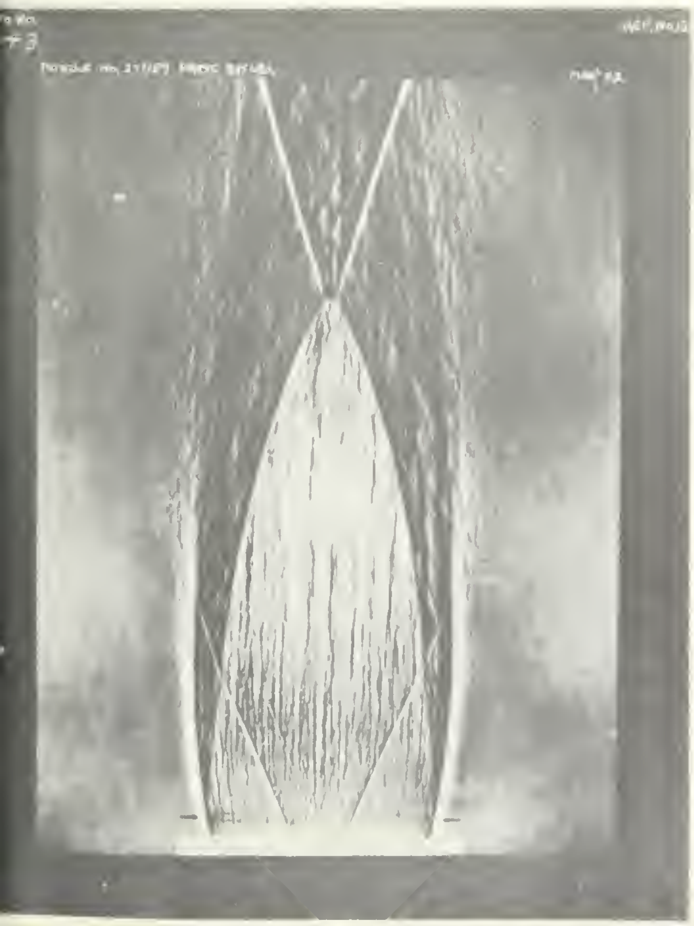


TABLE 3



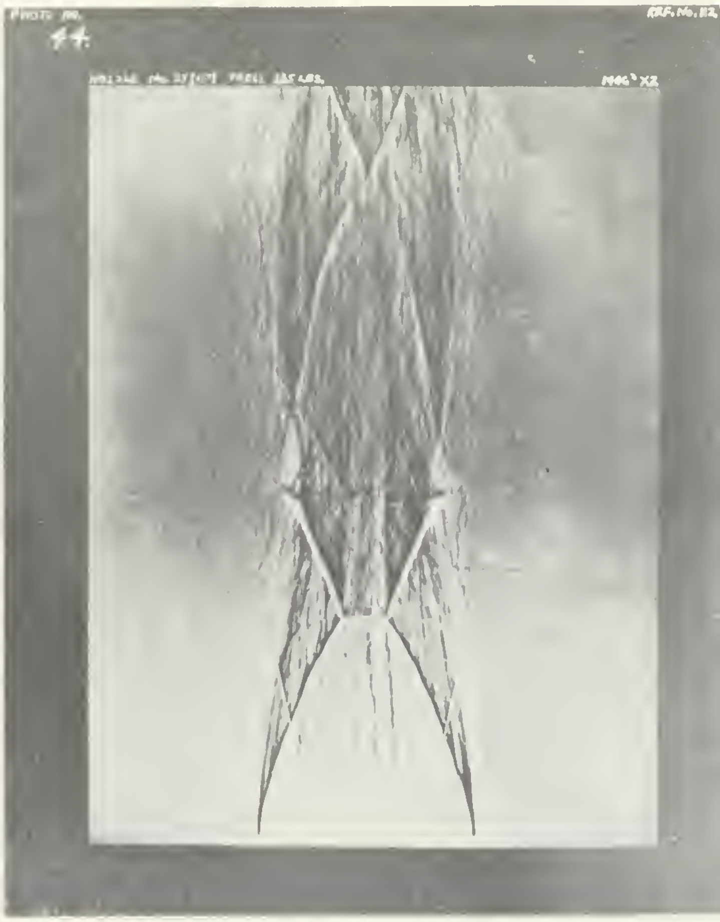
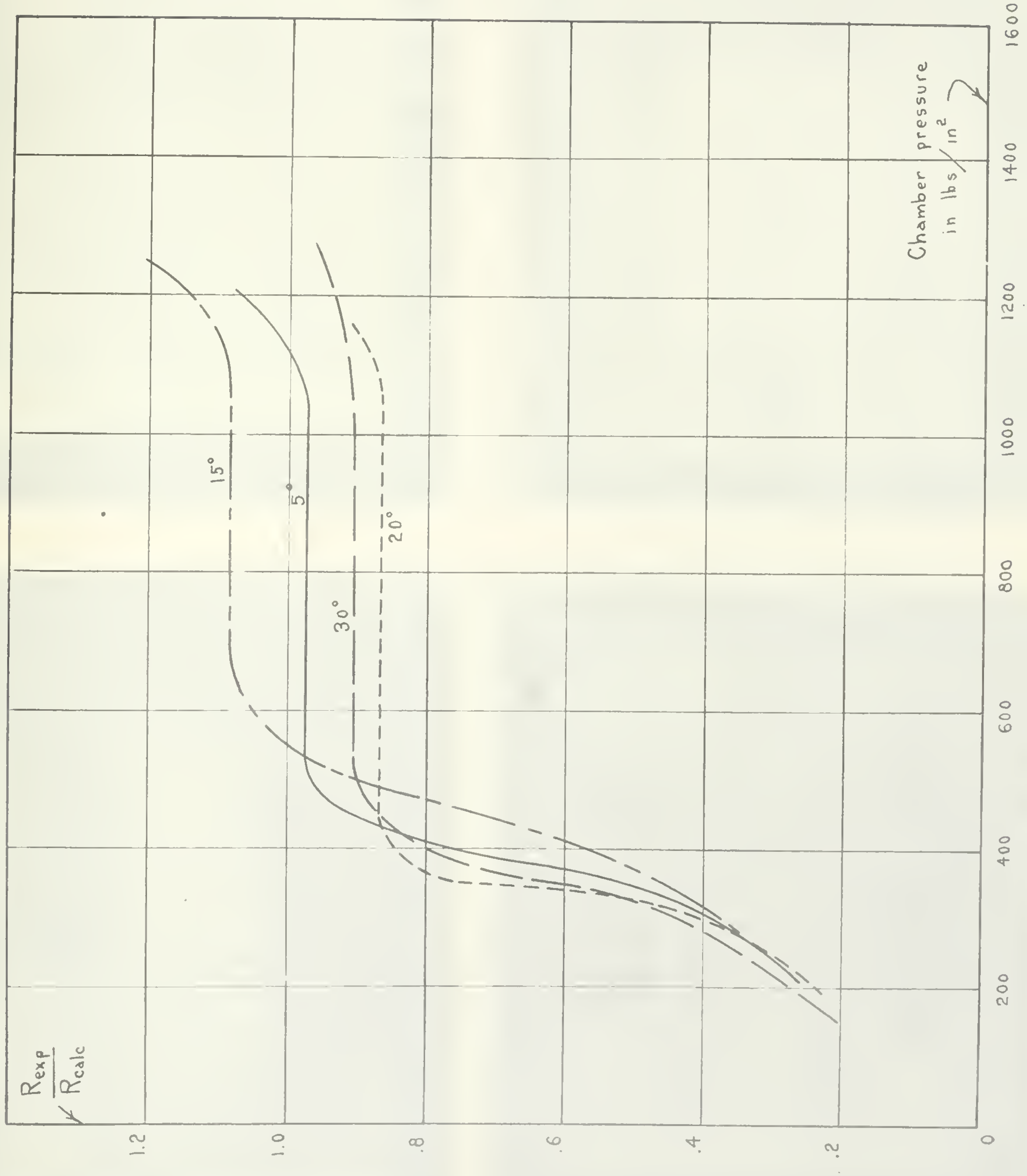


TABLE 4

TABLE 5



Chamber pressure in lbs/in²
R_{exp}/R_{calc}
0
200
400
600
800
1000
1200
1400
1600
.2
.4
.6
.8
1.0
1.2
15°
5°
30°
20°

TABLE 6

TABLE 7